



Setting up Equations – Translating Word Problems

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Translation Steps to Set up Equations

The hardest thing about doing word problems is taking the English words and translating them into mathematics. Usually, once the math equation is created, the actual math involved is often fairly simple. But figuring out the actual equation can seem nearly impossible.

Here is a list of hints and helps. Be advised, however: To *really* learn "how to do" word problems, requires practice, practice, practice!

- The first step to effectively translating and solving word problems is to read the problem entirely. Don't start trying to solve anything when you've only read half a sentence. Try first to get a feel for the whole problem; try first to see what information you have, and what you still need.

- The second step is to work in an organized manner. Figure out what you need but don't have, and name things. Pick variables to stand for the unknowns, clearly labeling these variables with what they stand for. Draw and label pictures neatly. Explain your reasoning as you go along. And make sure you know just exactly what the problem is actually asking for. You need to do this for two reasons:
 1. Working clearly will help you think clearly, and
 2. Figuring out what you need will help you translate your final answer back into English.

- The third step is to look for "key" words. Certain words indicate certain mathematical operations. Below is a partial list of key words and their mathematical equivalences.



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Mathematical Meaning	English Words	Mathematical Meaning	English Words
Addition	increased by more than combined together total of sum added to	Multiplication	of times multiplied by increased/decreased by a factor of (this type can involve both addition <i>and</i> multiplication!)
Subtraction	decreased by minus difference between less than fewer than	Division	per, a out of ratio of quotient of percent (divide by 100)
Equals	is, are, was, were, will be gives yields sold for		

Note that "per" means "divided by", as in "I drove 90 miles on three gallons of gas, so I got 30 miles per gallon".

Also, "a" sometimes means "divided by", as in "When I tanked up, I paid \$12.36 for three gallons, so the gas was \$4.12 a gallon".

Warning: The "less than" construction is backwards in the English from what it is in the math. If you need to translate "1.5 less than x ", the temptation is to write " $1.5 - x$ ". *Do not do this!* You can see how this is wrong by using this construction in a "real world" situation: Consider the statement, "He makes \$1.50 an hour less than me." You do not figure his wage by subtracting your wage from \$1.50. Instead, you subtract \$1.50 from your wage. So remember; **the "less than" construction is backwards.**

Also note that order is important in the "quotient/ratio of" and "difference between/of" constructions. If a problems says "the ratio of x and y ", it means " x divided by y ", not " y divided by x ". If the problem says "the difference of x and y ", it means " $x - y$ ", not " $y - x$ ".



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Extracting Key Words

Examples:

- Translate "the sum of 8 and y " into an algebraic expression.

This translates to " $8 + y$ "

- Translate "4 less than x " into an algebraic expression.

This translates to " $x - 4$ "

Remember: "Less than" is *backwards* in the math from how you say it in words!

- Translate " x multiplied by 13" into an algebraic expression.

This translates to " $13x$ "

- Translate "the quotient of x and 3" into an algebraic expression.

This translates to " $x/3$ "

- Translate "the difference of 5 and y " into an algebraic expression.

This translates to " $5 - y$ "

- Translate "the ratio of 9 more than x to x " into an algebraic expression.

This translates to " $(x + 9) / x$ "

- Translate "nine less than the total of a number and two" into an algebraic expression, and simplify.

This translates to " $(n + 2) - 9$ ", which then simplifies to " $n - 7$ "



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Extracting Multiple Key Words

Examples:

- **The length of a football field is 30 yards more than its width. Express the length of the field in terms of its width w .**

Whatever the width w is, the length is 30 more than this. Recall that "more than" means "plus that much", so you'll be adding 30 to w .

Answer: " $w + 30$ ".

- **Twenty gallons of crude oil were poured into two containers of different sizes. Express the amount of crude oil poured into the smaller container in terms of the amount g poured into the larger container.**

There are twenty gallons total, and we've already poured g gallons of it. How many gallons are left? There are $20 - g$ gallons left.

Answer: " $20 - g$ ".

"How Much Is Left" Problems

These problems will provide some total amount. Smaller amounts, of unspecified sizes, are added (combined, mixed, etc) to create this total amount. Pick a variable to stand for one of these unknown amounts. After having thus accounted for one of the amounts, the remaining amount is whatever is left after deducting this named amount from the total.

- Some problems might say that a trip took ten hours, and that the trip had two legs. You might name the time for the first leg as " t ", with the *remaining* time for the second leg being $10 - t$.
- Other problems might say that a hundred-pound order of animal feed was filled by mixing products from Bins A, B, and C, and that twice as much was added from Bin C as from Bin A. Let " a " stand for the amount from Bin A. Then the amount from Bin C was " $2a$ ", and the amount taken from Bin B was the remaining portion of the hundred pounds: $100 - a - 2a$.



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Number Problems

"Number" problems are fairly contrived, but they're also fairly standard. After all, the point of these problems isn't their relation to "real life", but your ability to extract the math from the English.

Examples:

- **The sum of two consecutive integers is 15. Find the numbers.**

The problem gives two pieces of information. First, I know that I am adding two numbers, and their sum is fifteen. Second, I know that the numbers are nice neat round numbers (like -3 or 6), not messy ones (like -4.628 or $17/32$), and that the second number is one more than the first. This last piece of information comes from the fact that "consecutive integers" (or "consecutive whole numbers", if they're restricting the possibilities to only positive numbers) are one unit apart. Examples of "consecutive integers" would be -12 and -11 , 1 and 2 , and 99 and 100 .

Represent the first number by " n ". Then the second number has to be " $n + 1$ ". Their sum is then:

$$n + (n + 1) = 15$$

$$2n + 1 = 15$$

$$2n = 14$$

$$n = 7$$

The problem did not ask for the value of the variable n ; it asked for the identity of two numbers. So the answer is not " $n = 7$ "; the actual answer is:

"The numbers are 7 and 8."



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- **The product of two consecutive negative even integers is 24. Find the numbers.**

The problem tells quite a bit about these two numbers: the numbers are even and they are negative. (The fact that they are negative may help if we find two solutions — a positive and a negative — then we will know which one to pick.) Since even numbers are two apart (for example, -4 and -2 or 10 and 12), then we also know that the second number is two greater than the first. When we multiply the two numbers, the answer will be 24 . In other words, letting the first number be " n " and the second number be " $n + 2$ ":

$$\begin{aligned}(n)(n + 2) &= 24 \\ n^2 + 2n &= 24 \\ n^2 + 2n - 24 &= 0 \\ (n + 6)(n - 4) &= 0\end{aligned}$$

Then the solutions are $n = -6$ and $n = 4$. Since the problem defined the numbers in question as negative, ignore the " 4 " and take $n = -6$. Then the next number is $n + 2 = -4$, and the answer is:

“The numbers are -6 and -4 .”

- **Twice the larger of two numbers is three more than five times the smaller, and the sum of four times the larger and three times the smaller is 71. What are the numbers?**

The first step in this problem is to label everything:

the larger number: x
the smaller number: y

twice the larger: $2x$
three more than five times the smaller: $5y + 3$
relationship between ("is"): $2x = 5y + 3$

four times the larger: $4x$
three times the smaller: $3y$
relationship between ("sum of"): $4x + 3y = 71$



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Now we have two equations and two variables:

$$\begin{aligned}2x &= 5y + 3 \\4x + 3y &= 71\end{aligned}$$

Solve the first equation for x :

$$x = (5/2)y + (3/2)$$

Then plug the right-hand side of this equation into the second equation in place of the "x":

$$\begin{aligned}4[(5/2)y + (3/2)] + 3y &= 71 \\10y + 6 + 3y &= 71 \\13y + 6 &= 71 \\13y &= 65 \\y &= 65/13 = 5\end{aligned}$$

Now that we have solved for y , we can solve for x :

$$\begin{aligned}x &= (5/2)y + (3/2) \\x &= (5/2)(5) + (3/2) \\x &= (25/2) + (3/2) \\x &= 28/2 = 14\end{aligned}$$

As always, remember to answer the question that was actually asked. The solution here is not " $x = 14$ ", but is the following sentence:

“The larger number is 14, and the smaller number is 5.”

The trick to doing this type of problem is to label everything very explicitly. Until you become used to doing these, do not attempt to keep track of things in your head. Clearly label every single step to work out these problems rather easily.

Reference: <http://www.purplemath.com/modules/>