Definition of the Quadratic Formula

The Quadratic Formula uses the a, b and c from $ax^2 + bx + c = 0$, where a, b and c are just numbers; they are the "numerical coefficients".

The Quadratic Formula is:

For $ax^2 + bx + c = 0$, the value of x is given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

What are you finding by using the Quadratic Formula?

Suppose you have $ax^2 + bx + c = y$, and you are told to plug zero in for y. The corresponding x-values are the x-intercepts of the graph. So solving $ax^2 + bx + c = 0$ for x means, among other things, that you are trying to find x-intercepts. If there are two solutions for the quadratic, there must then be two x-intercepts on the graph.

Tips for Using the Quadratic Formula

Often, the simplest way to solve for the value of x when you are given an equation similar to $ax^2 + bx + c = 0$ is to factor the quadratic, set each factor equal to zero, and then solve each factor. But sometimes the quadratic is too messy, or it doesn't factor at all, or you just don't feel like factoring.
While factoring may not always be successful, the Quadratic Formula can *always* find the solution.

For the Quadratic Formula to work, you *must* have your equation arranged in the form "(quadratic) = 0".

Also, the "2a" in the denominator of the Formula is underneath *everything* above, not just the square root. And it's a "2a" under there, not just a plain "2".

Make sure that you are careful not to drop the square root or the "plus/minus" in the middle of your calculations.

Remember that "b^2" means "the square of ALL of b, including its sign", so don't leave b^2 being negative, even if b is negative, because the square of a negative is a positive.

Examples using the Quadratic Formula

Solve:

a) $x^2 + 3x - 4 = 0$

Using $a = 1$, $b = 3$, and $c = –4$, the solution looks like this:

$$
\begin{aligned}
x &= \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-4)}}{2(1)} \\
&= \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm \sqrt{25}}{2} \\
&= \frac{-3 \pm 5}{2} = \frac{-3-5}{2}, \frac{-3+5}{2} \\
&= \frac{-8}{2}, \frac{2}{2} = -4,\ 1 = x
\end{aligned}
$$

The solution is $x = –4$, $x = 1$. (Note: This quadratic is factorable.)

b) $2x^2 - 4x - 3 = 0$

In this case, $a = 2$, $b = –4$, and $c = –3$:

$$
\begin{aligned}
x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)} \\
&= \frac{4 \pm \sqrt{16+24}}{4} = \frac{4 \pm \sqrt{40}}{4} = \frac{4 \pm \sqrt{4}\sqrt{10}}{4} \\
&= \frac{4 \pm 2\sqrt{10}}{4} = \frac{2\left(2 \pm \sqrt{10}\right)}{2(2)} = \frac{2 \pm \sqrt{10}}{2} \\
&= \frac{2-\sqrt{10}}{2}, \frac{2+\sqrt{10}}{2} \\
&\approx -0.58113883,\ \ 2.5811388 = x
\end{aligned}
$$

Warning: The "solution" or "roots" or "zeroes" of a quadratic are usually required to be in the "exact" form of the answer. In the example above, the exact form is the one with the square roots of ten in it. Unless the directions say to round the answer, leave it in the exact form.

Answer: $= \frac{2-\sqrt{10}}{2}, \frac{2+\sqrt{10}}{2}$

Using the Quadratic Formula to find the x-intercepts

Example: Let $x^2 + 3x - 4 = 0$

Using $a = 1$, $b = 3$, and $c = -4$, the solution looks like this:

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-4)}}{2(1)}$$

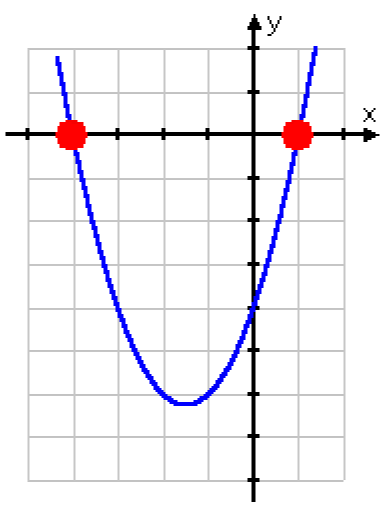
$$= \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm \sqrt{25}}{2}$$

$$= \frac{-3 \pm 5}{2} = \frac{-3-5}{2}, \frac{-3+5}{2}$$

$$= \frac{-8}{2}, \frac{2}{2} = -4, 1 = x$$

The solution is $x = -4$, $x = 1$.

Since there were two solutions for $x^2 + 3x - 4 = 0$, there must then be two x-intercepts on the graph. Graphing, we get the curve below:



As you can see, the x-intercepts match the solutions, crossing the x-axis at $x = -4$ and $x = 1$. This shows the connection between graphing and solving: When you are solving "(quadratic) = 0", you are finding the x-intercepts of the graph.

More Examples using the Quadratic Formula

Examples:

Solve:

a) $x(x - 2) = 4.$

First: Rearrange the equation in the form "(quadratic) = 0"

$$x(x - 2) = 4$$
$$x^2 - 2x = 4$$
$$x^2 - 2x - 4 = 0$$

Next: Using *a* = 1, *b* = –2, and *c* = –4, the solution looks like this:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4+16}}{2} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm \sqrt{4}\sqrt{5}}{2}$$
$$= \frac{2 \pm 2\sqrt{5}}{2} = \frac{2(1 \pm \sqrt{5})}{2(1)} = 1 \pm \sqrt{5}$$
$$\approx -1.236068, \quad 3.236968 = x$$

Answer: $\mathbf{x = 1 \pm \sqrt{5}}$

b) $9x^2 + 12x + 4 = 0.$

Using *a* = 9, *b* = 12, and *c* = 4, the solution looks like this:

$$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(9)(4)}}{2(9)}$$
$$= \frac{-12 \pm \sqrt{144-144}}{18} = \frac{-12 \pm \sqrt{0}}{18}$$
$$= \frac{-12 \pm 0}{18} = \frac{-12}{18} = \frac{6(-2)}{6(3)} = -\frac{2}{3} = x$$

Answer: $\mathbf{x = {}^{-2}/_{3}}$

In the previous examples, there were two solutions because of the "plus-minus" part of the Formula. In this case, though, the square root reduced to zero, so the plus-minus didn't count for anything.

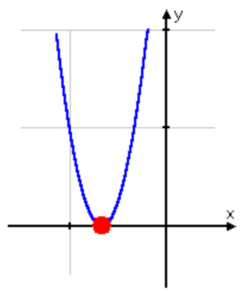
This solution is called a "repeated" root, because *x* is equal to $^{-2}/_{3}$, but it is equal two ways: $^{-2}/_{3} + 0$ and $^{-2}/_{3} - 0$.

You can also see this repetition better if you factor:

$9x^2 + 12x + 4 = (3x + 2)(3x + 2) = 0$, so $x = {}^{-2}/_{3}$ and $x = {}^{-2}/_{3}$.

Anytime you get zero in the square root of the Quadratic Formula, you'll only get one solution.

This is what the graph looks like:



The parabola just touches the *x*-axis at $x = {}^{-2}/_{3}$; it doesn't actually cross. This is always true: if you have a root that appears exactly twice, then the graph will touch the axis, but not pass through.

Answers Containing No Real Solutions

Solve: $3x^2 + 4x + 2 = 0$.

Using $a = 3$, $b = 4$, and $c = 2$, the solution looks like this:

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{16 - 24}}{6} = \frac{-4 \pm \sqrt{-8}}{6} = x$$

At this point, there is a negative number inside the square root. Since this learning tool does not cover complex numbers, the solution stops here.

Answer: No real solution.

Reference: http://www.purplemath.com/modules/quadform.htm