



Pre-Calculus II

Factoring and Operations on Polynomials



Factoring.....	1
Polynomials.....	1
Addition of Polynomials.....	1
Subtraction of Polynomials.....	1
Multiplication of Polynomials.....	2
Multiplying a monomial by a monomial.....	2
Multiplying a monomial by a polynomial.....	2
Multiplying a polynomial by a polynomial.....	2
Factoring polynomials that have a common monomial factor.....	3
Factoring equations of the form x^2+bx+c	3
Factoring equations of the form $ax^2 + bx +c$	3
Division of Polynomials.....	4
Long Division.....	4
Synthetic Division.....	5



Pre-Calculus II

Factoring and Operations on Polynomials



Polynomials

A polynomial is usually described by the number of terms it contains and its degrees.

Different polynomials are described based on the number of terms.

Number of terms	Name	Example
1	Monomial	$3x$, $6(\text{constant})$
2	Binomial	$2x+4$, $2x^2+4x$
3	Trinomial	$2x^2+4x+6$
4 or more	Polynomial	$3x^3+4x^2+2x+7$

Coefficient is a constant by which a variable is multiplied.

Leading coefficient is the coefficient of the leading (highest) term in a given polynomial.

The **degree of the polynomial** is equal to the highest degree (of any term) in the polynomial.

The first three are special polynomials.

Example 1: The degree of $3t$ is 1

Example 2: $4x^4+3x^2+2x$

- Degree of the given polynomial is 4
- The coefficient of: x^4 is 4, x^2 is 2, and x is 2
- The leading coefficient is 4

Note: The degree of constant is zero

Addition of Polynomials

1. Combine all the like terms.
2. Add all the like terms together.

Note: Like terms must contain the same variable(s) raised to the same power(s).

Example: Add $(3x^4+2x^2+3x)$ and $(5x^3+6x^2+3)$

$$\begin{aligned} \text{Solution: } & (3x^4+2x^2+3x) + (5x^3+6x^2+3) \\ & = 3x^4+5x^3+2x^2+6x^2+3x+3 \\ & = 3x^4+5x^3+8x^2+3x+3 \end{aligned}$$

Try this: Add $(5ab+3ab^2+7a)$ and $(8b+6a^2b+2ab+3a)$

Subtraction of Polynomials

1. Place parentheses around the expression being subtracted.
2. Remove the parentheses by distributing the minus sign. This will change the sign of each term of the polynomial being subtracted.
3. Add the polynomial by combining like terms.

Example: Subtract $(3x^2+2x+3)$ from $(5x^2+6x+2)$

$$\begin{aligned} \text{Solution: Step1: } & 5x^2+6x+2-(3x^2+2x+3) \\ \text{Step2: } & 5x^2+6x+2-3x^2-2x-3 \\ \text{Step3: } & 5x^2-3x^2+6x-2x+2-3 \\ & = 2x^2+4x-1 \end{aligned}$$



Pre-Calculus II

Factoring and Operations on Polynomials



Try this: Subtract $(-2a^2 - 6a + 10)$ from $(a^2 + 8)$

Multiplication of Polynomials

Multiplying a Monomial by a Monomial

1. Multiply the numerical coefficients to find the coefficient of the product.
2. Multiply the variables, simplifying powers with the same base.

Example: Multiply $(4x)$ by $(3x^2)$

Solution: $4x * 3x^2 = (4*3)(x*x^2) = 12x^3$

Try This: Multiply $(3a^3b^2)$ by $(-2ab)$

Multiplying a Monomial by a Polynomial

Distribute the monomial over the polynomial by multiplying the monomial by every term in the polynomial and joining the products by their proper sign.

Example: Multiply $(3x - 4)$ by 5

Solution: $5(3x - 4) = 15x - 20$

Multiplying a Polynomial by a Polynomial

Multiply each term of the first polynomial by each term of the second polynomial, then combine like terms.

Example: Multiply $(5x-7)$ by $(6x + 4)$

Solution: $5x * 6x + 5x * 4 - 7 * 6x - 7 * 4 = 30x^2 + 20x - 42x - 28 = 30x^2 - 22x + 28.$

Try this: Multiply $(3x^2 + 2x)$ by $(4x + 3)$

Factoring

A product is found by multiplying numbers or terms together. The numbers or terms multiplied together are called the factors of the product. A factor may consist of more than one term.

Example: $5(x+y) = 5x + 5y$

One factor is 5 and second is $(x+y)$

Factoring Polynomials with a Common Monomial Factor

1. Factor out the greatest common factor(GCF) from each term of the polynomial.
2. Write the two factors as the indicated product.

Example: $3a^4 + 6a^3 + 3a^2$

Solution: GCF is $3a^2$

$$3a^2(a^2+3a+1)$$

Try This: Factor $(a^2b^2 - ab)$

Factoring Equations of the Form x^2+bx+c

Example: Factor $x^2 + 7x + 12$



Pre-Calculus II

Factoring and Operations on Polynomials



Solution: The first term in each binomial factor is x.

$$x^2 + 7x + 12$$

$$(x \quad)(x \quad)$$

Search for two factors of 12 that have a sum of 7

Factors of 12

(1)(12)

(2)(6)

(3)(4) \longrightarrow These are the factors of 12 that have the sum as 7.

Choose 3 and 4 since $3 \cdot 4 = 12$ and $3 + 4 = 7$

Factor by using the correct combination of terms

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Final answer is $(x + 3)(x + 4)$

Check: $(x + 3)(x + 4) = x^2 + 7x + 12$

Try this: Factor $x^2 - 5x + 6$

Factoring Equations of the Form $ax^2 + bx + c$

Example: Factor $5x^2 + 21x + 4$

Solution: Write all possible factors of 5 and 4

$$5x^2 + 21x + 4$$

For 5

1 * 5

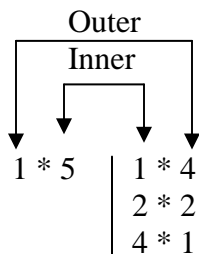
For 4

1 * 4

2 * 2

4 * 1

Search for the combination that will yield a sum of 21.



Outer	Inner	Sum
1 * 4	5 * 1	9
1 * 2	5 * 2	12
1 * 1	5 * 4	21 * STOP *

Since the correct combinations are 1 and 5, and 4 and 1, we can now factor accordingly.

$$5x^2 + 21x + 4$$

$$1 * 5 \quad 4 * 1$$

$$\downarrow \quad \swarrow \quad \downarrow$$

$$(x \ 4) \ (5x \ 1)$$

$$(x + 4) (5x + 1)$$

Final answer is $(x + 4)(5x + 1)$

Try This: Factor: $4x^2 - 12x + 9$



Pre-Calculus II

Factoring and Operations on Polynomials



Division of Polynomials

Divisor could be defined as a quantity or variable that divides another. **Dividend** is simply the number or quantity that is to be divided.

For example, when we divide $(5x-7)$ by $(6x + 4)$, $(6x+4)$ is the divisor and $(5x-7)$ is the dividend.

Long Division

1. Write the problem in long division form.
 - a. Arrange the divisor and dividend in descending order.
 - b. Account for missing terms.
2. To find the terms of the quotient:
 - a. Divide the first term in the divisor into the first term in the dividend; write the answer as the first term in the quotient.
 - b. Multiply the divisor by the first term in the quotient, writing the product under the dividend.
 - c. Subtract like terms.
 - d. Bring down the next term.
3. Repeat (2) divide, multiply, subtract and bring down.
4. Continue until the remainder has a lower degree than divisor.

Example: Divide $(x^2 + 5x + 6)$ by $(x + 2)$

Solution: 1. Write in long division form

$$x + 2 \overline{)x^2 + 5x + 6}$$

2. Find the first term in the quotient, $x + 2 \overline{)x^2 + 5x + 6}$

a. What must you multiply 'x' by to get 'x²'?

b. Multiply (x) times (x+2)

$$x + 2 \overline{)x^2 + 5x + 6}$$

$$x^2 + 2x$$

c. Subtract like terms

$$x + 2 \overline{)x^2 + 5x + 6}$$

$$-(x^2 + 2x)$$

$$3x$$

d. Bring down the next term

$$x + 2 \overline{)x^2 + 5x + 6}$$

$$-(x^2 + 2x)$$

$$3x + 6$$

3. Find the second term in the quotient,



Pre-Calculus II

Factoring and Operations on Polynomials



a. Divide x into $+3x$

$$\begin{array}{r} x+3 \\ x+2 \overline{) x^2 + 5x + 6} \\ \underline{-(x^2 + 2x)} \\ 3x + 6 \end{array}$$

b. Multiply $+3$ times $(x+2)$

$$\begin{array}{r} x+3 \\ x+2 \overline{) x^2 + 5x + 6} \\ \underline{-(x^2 + 2x)} \\ 3x + 6 \\ 3x + 6 \end{array}$$

c. Subtract like terms

$$\begin{array}{r} x+3 \\ x+2 \overline{) x^2 + 5x + 6} \\ \underline{-(x^2 + 2x)} \\ 3x + 6 \\ \underline{-(3x + 6)} \\ 0 + 0 \end{array}$$

Final Answer is: $(x+3)$

Try this: Divide (x^3+8) by $(x+2)$

Synthetic Division

Synthetic division is a short method of polynomial division. The following example will explain better about synthetic division.

Example: Divide $x^2 + 5x + 6$ by $(x-1)$

- For the divisor, write $(x-1) = 0$
 $x-1 = 0$
 $x = 1$

- Write the coefficients of the dividend $x^2 + 5x + 6$ in the following manner.

$$\begin{array}{r|rrr} & 1 & 5 & 6 \\ \hline & & & \end{array}$$

- Now write the value $x=1$ from step1 to the left.

$$\begin{array}{r|rrr} 1 & 1 & 5 & 6 \\ \hline & & & \end{array}$$



Pre-Calculus II

Factoring and Operations on Polynomials



4. Carry down the first number from the left (leading coefficient).

$$\begin{array}{r|rrr}
 1 & 1 & 5 & 6 \\
 & \downarrow & & \\
 & 1 & &
 \end{array}$$

5. Multiply this number by 1 (value of x written on the left) and write the result on the next column.

$$\begin{array}{r|rrr}
 1 & 1 & 5 & 6 \\
 \swarrow \text{Multiply} & & & \\
 & 1 & & 1
 \end{array}$$

6. Add down the column.

$$\begin{array}{r|rrr}
 1 & 1 & 5 & 6 \\
 \swarrow & & \downarrow & \\
 & 1 & 1 & 6
 \end{array}$$

7. Multiply the result by 1 (value of x on the left) and write the result in the next column.

$$\begin{array}{r|rrr}
 1 & 1 & 5 & 6 \\
 \swarrow & & & \\
 & 1 & 6 & 6
 \end{array}$$

8. Add down the column.

$$\begin{array}{r|rrr}
 1 & 1 & 5 & 6 \\
 & & & \downarrow \\
 & & 1 & 6 \\
 \hline
 1 & 6 & & 12 \rightarrow \text{Remainder}
 \end{array}$$

This should be recognized as $(1x + 6)$ which is our quotient and the number 12 to the right is always the remainder.

Final Answer: Quotient = $(x + 6)$
 Remainder = 12

Try this: Divide $2x^2 + 3x - 8$ by $(x-3)$