Linear Functions

A function of the form f(x) = *m*x + *b* where m is a nonzero number and m and b are real numbers, is a **linear function** of x.

Example: f(x) = 2x + 2

Finding the Slope of a Line

A **slope** *m* of the line passing through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ with $x_1 \neq x_2$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The lines that have a positive slope slant upward from left to right. The lines that have a negative slope slant downward from left to right.

Example: Find the slope of the line passing through the points whose coordinates are given as (-3, 4) and (1, -2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{1-(-3)} = \frac{-6}{4} = -\frac{3}{2}$$

Since m < 0 (negative), the line slants downwards from left to right.

Slope-Intercept Form

The equation f(x) = m*x* + b is called the slope intercept form of the equation of a line because the graph f(x) = m*x* + b is a line with slope m and y-intercept (0, b).

Example: f(x)= 2x – 1

The equation y=2x-1 is in slope intercept form, with b=-1 and m=2. Thus the y-intercept is (0, -1), and the slope is 2.

$$m = \frac{2}{1} = 2$$

Point Slope Form

We can find an equation of a line provided we know its slope and at least one point on the line. If (x_1, y_1) is a point on a line of slope m, and (x, y) is any other point on the line, then

$$\frac{y - y_1}{x - x_1} = m \text{ where } x \neq x_1$$

Which is also **$(y - y_1) = m(x - x_1)$**

Example: Find an equation of the line with slope -3 that passes through (-1, 4)

Use the point slope form with m = -3, x_1= -1 and y_1= 4
$y - y_1 = m(x - x_1)$
y- 4 = 3[x-(-1)]

y – 4 = -3x – 3
y = -3x + 1

Parallel Lines

Two non-vertical lines are parallel if and only if their slopes are equal and they have different *y*-intercept.

Let Line L_1 represent $f_1(x) = m_1x + b$ and L_2 represent $f_2(x) = m_2x + b$. Then, lines L_1 and L_2 are **parallel** if and only if $m_1 = m_2$

Example: $f_1(x) = 3x + 1$ and $f_2(x) = 3x - 4$.

Here, $m_1 = m_2 = 3$. Hence the lines are parallel.

Line Parallel to a Given Line

Example: Find an equation of the line that has the point (2, -3) and is parallel to the line $2x+y=6$.

1. Write the given equation in slope-intercept form.

   $2x + y = 6$

   $y = -2x + 6$ *Solve for 'y' to bring it to the form $y = mx + b$*

2. Since the 2 lines are parallel, the slope of the line we need to find is the same as the slope of the given line. The slope of the given line is -2.

   Therefore the slope of the parallel line is -2

3. Use the point – slope form to find the equation of parallel line that has the point (2,-3)

   $y - y_1 = m(x - x_1)$ *Point- slope form*

   $y + 3 = -2(x - 2)$ *$m=-2$, $x_1= 2$, $y_1 = -3$*

   $y + 3 = -2x + 4$

   $y = -2x + 1$ *Slope intercept form*

   $2x+y = 1$ ***Equation of the line parallel to the given line***

Perpendicular Lines

Two non-vertical lines are perpendicular if and only if the product of their slopes is -1.

Let Line L_1 represent $f_1(x) = m_1x + b$ and L_2 represent $f_2(x) = m_2x + b$. Then, lines L_1 and L_2 are **perpendicular** if and only if $m_1 = -\frac{1}{m_2}$ *or* $m_2 = -\frac{1}{m_1}$

Example: $f_1(x) = 3x + 1$ and $f_2(x) = -\frac{1}{3}x - 4$.

Here, $m_1 = -\frac{1}{m_2} = 3$. Hence the lines are perpendicular.

Line Perpendicular to a Given Line

Example: Find an equation with the point (1,-2) that is perpendicular to the line $x + 3y = 6$.

1. Write the given equation in slope- intercept form

   $x + 3y = 6$

   $3y = -x + 6$ *Solve for 'y' to bring it to the form $y = mx + b$*

   $y = -\frac{1}{3}x + 2$

2. Since the 2 lines are perpendicular, the slope of the line we need to find and the slope of the given line has a product equal to -1. The slope of the given line is $m_1 = -\frac{1}{3}$. Let the slope of the perpendicular line be m_2.

   Then $m_1.m_2 = -1$

   $-\frac{1}{3}.m_2 = -1$

   $m_2 = 3$

   Therefore slope of the perpendicular line is 3.

3. Use point-slope form to find the equation of the perpendicular line that has the point (1, - 2).

   $y - y_1 = m(x - x_1)$ *Point- slope form*

   $y - (-2) = 3(x - 1)$ *$m=3, y_1=-2, x_1=1$*

   $y + 2 = 3x - 3$

   $y = 3x - 5$ *Slope-intercept form*

   $\mathbf{3x - y = 5}$ ***Equation of the line perpendicular to the given line***

Quadratic Equations

A quadratic function of x is a function that can be represented by an equation of the form,

$$\boldsymbol{f(x) = a x^2 + bx + c}, \quad \text{where } a, b, \text{ and } c \text{ are real numbers and } a \neq 0$$

The graph of the above function is a parabola.

Solving Quadratic Equations by Factoring

1. Write the equation in standard form.
2. Factor the quadratic expression completely, so that the factors are only binomials and constants. [If there is a trinomial as one of the factors which cannot be factored anymore then, you have to solve the equation using the Quadratic Formula, which we'll be seeing later.]

3. Set each factor found in (2) equal to zero and solve each equation for the variable.
4. Check all solutions by using the original equation.

Example: Solve $4x^2 + 10x = 6$
Solution: $4x^2 + 10x - 6 = 0$

$2(2x^2 + 5x - 3) = 0$
$2(2x - 1)(x + 3) = 0$
$(2x - 1)(x + 3) = 0$
$2x - 1 = 0 \qquad x + 3 = 0$
$2x = 1 \qquad x = -3$
$x = ½$
Final answer is $x = ½$, $x = -3$

Try This: Solve $x^2 - 7x = 0$

Solving Quadratic Equations by Completing the Square

1. Transform the equation into the form $x^2 + bx = c$, where the coefficient of x^2 is one.
2. Complete the square.
   a. Take one half the coefficient of x, i.e., b/2
   b. Square this number, i.e., $(b/2)^2$
   c. Add this value to each side of the equation, i.e., $x^2 + bx + (b/2)^2 = c + (b/2)^2$
3. Factor the perfect square trinomial into a binomial squared and combine like terms.
4. Solve by using the square root method.
5. Check the solution(s) in the original equations.

Example: Solve $x^2 + 10x - 7 = 0$ by completing the square.
Solution: Isolate the constant term.

$x^2 + 10x - 7 = 0$
$x^2 + 10x = 7$

Complete the square by adding $[1/2\ (10)]^2 = 5^2 = 25$ to both sides of the equation.

$x^2 + 10x = 7$
$x^2 + 10x + 25 = 7 + 25$
$(x+5)^2 = 32$

Solve by using the square root method.

$(x + 5)^2 = 32$

$$\sqrt{(x+5)^2} = \pm\sqrt{32}$$
$$x + 5 = \pm\sqrt{16}\sqrt{2}$$
$$x + 5 = \pm 4\sqrt{2}$$
$$x + 5 = 4\sqrt{2} \qquad x + 5 = -4\sqrt{2}$$
$$x = 4\sqrt{2} - 5 \qquad x = -4\sqrt{2} - 5$$

Final answer: $x = 4\sqrt{2} - 5$; $x = -4\sqrt{2} - 5$

Try this: Solve $x^2 + 6x - 1 = 0$

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Solving Quadratic Equations by the Quadratic Formula

1. Express the quadratic equation in standard form ($ax^2 + bx + c = 0$).
2. Identify the values for a, b and c.
3. Replace the values for a, b and c directly into the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Use parentheses to maintain form.
4. Perform the indicated operations.
5. Simplify any resulting radicals.
6. Check the solution(s) in the original equation.

Example: Solve $x^2 - 4x + 4 = 0$ by the quadratic formula.
Solution: With the equation in standard form, we identify the values for a, b, and c.

a = 1, b = -4, c =4

Substitute these values directly into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 16}}{2}$$

$$x = \frac{4 \pm \sqrt{0}}{2}$$

$$x = \frac{4}{2}$$

$$x = 2$$

Final answer: x =2

Try this: $2x^2 + 6x = 9$ by the quadratic formula.