What is a Fraction?

A fraction is a number that describes part of a whole. Fractions are written in the form

$$\frac{a}{b}$$

where a is called the numerator and b is called the denominator.
The denominator of a fraction shows the number of equal parts in a whole. The numerator of a fraction is the number of parts being used or talked about.

Example: Use a fraction to represent the shaded part of the object.

Solution: The object is made up of 10 equal blocks. This means that the denominator of the fraction is 10. The number of blocks that are shaded is 7, thus the numerator of the fraction is 7.

$$\frac{7}{10}$$

Reducing Fractions

Fractions should always be left in lowest terms. This means that the numerator and denominator have no common factors, or they are relatively prime.

To reduce a fraction, follow the following steps:
Step 1) Factor the numerator.
Step 2) Factor the denominator.
Step 3) Cancel any factors that appear in both the numerator and the denominator.

Example: Reduce the following fractions:

a) $\frac{24}{36}$ b) $\frac{60}{144}$ c) $\frac{5}{15}$

Solutions:

a) $\frac{24}{36} = \frac{2 \cdot 12}{3 \cdot 12} = \frac{2 \cdot \cancel{12}}{3 \cdot \cancel{12}} = \frac{2}{3}$

b) $\frac{60}{144} = \frac{5 \cdot 12}{12 \cdot 12} = \frac{5 \cdot \cancel{12}}{12 \cdot \cancel{12}} = \frac{5}{12}$

c) $\frac{5}{15} = \frac{5 \cdot 1}{5 \cdot 3} = \frac{\cancel{5} \cdot 1}{\cancel{5} \cdot 3} = \frac{1}{3}$

Special Notes

Let a be a nonzero integer.

1. If the numerator and denominator of a fraction are the same, then the fraction reduces to 1.

$$\frac{a}{a} = 1$$

2. If the denominator of a fraction is 1, then the fraction reduces to the numerator.

$$\frac{a}{1} = a$$

3. If the numerator of a fraction is 0 and the denominator is not 0, then the fraction reduces to 0.

$$\frac{0}{a} = 0$$

4. If the denominator of a fraction is 0, then the fraction is undefined.

$$\frac{a}{0} = \text{undefined}$$

Equivalent Fractions

To find equivalent fractions, you multiply the numerator and the denominator of a given fraction by the same number.

Example: Find 3 fractions that are equivalent to $\frac{2}{3}$.

Solution: To solve this problem we will multiply the numerator and denominator 3 times by 3 different numbers.

$$\frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6} \qquad \frac{2 \cdot 3}{3 \cdot 3} = \frac{6}{9} \qquad \frac{2 \cdot 5}{3 \cdot 5} = \frac{10}{15}$$

So, 3 fractions equivalent to $\frac{2}{3}$ are $\frac{4}{6}, \frac{6}{9}$, and $\frac{10}{15}$. There are many other fractions.

Adding and Subtracting Fractions

A. Fractions with a common denominator

Steps

1. Add or subtract the numerators of the fractions
2. Carry the common denominator into the denominator of the new fraction
3. Reduce, if possible

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$
$$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$$

B. Fractions with different denominators

Finding the Least Common Denominator (LCD)

When the denominators of fractions are different, you must first find the a common denominator for the fractions. The new denominator will be the least common denominator, or LCD. The least common denominator is the smallest number which is a common multiple of each of the original denominators.

Steps for find the LCD of a set of fractions:

1. Write the prime factorization of each denominator
2. Note all prime factors that occur. For each prime factor that occurs, determine in which denominator it occurs the most. Write down the prime factor the number of times it occurs in that one denominator. (If prime factorizations are written in exponential form, take the prime with the highest exponent.)
3. Calculate the LCD of your fractions. To do this, multiply the factors written down in step 2.

Example: Find the LCD for the fractions $\frac{10}{24}$ and $\frac{22}{45}$.

Solution: First we find the prime factorizations of 24 and 45

$24 = 2^3 \cdot 3$ and $45 = 3^2 \cdot 5$

Now, the primes that we have are 2, 3, and 5. We notice that the highest power on 2 is 3, the highest power on 3 is 2, and the highest power on 5 is one. This gives us an LCD of:

$$LCD = 2^3 \cdot 3^2 \cdot 5 = 360$$

Steps for Adding or Subtracting Fractions with Different Denominators

1. Find the LCD for the fractions being added or subtracted
2. Find the equivalent fractions with the LCD in the denominator
3. Add or subtract the numerators of the fractions
4. Simplify the resulting fractions.

Example: Add $\frac{1}{2}+\frac{1}{6}$

Solution: Since the fractions do not have the same denominator, we must first find the LCD.
Since $6=2\cdot 3$, then 6 is the LCD.
Now, we must get the equivalent fractions:

For $\frac{1}{2}$, we need the denominator to be 6, so we must multiply 2 by 3 to get 6. In order to not change the problem, we must also multiply the numerator by 3 as well. Thus,

$$\frac{1\cdot 3}{2\cdot 3}=\frac{3}{6}$$

$\frac{1}{6}$ already has 6 as the denominator, so we do not need to change the fraction.

$$\frac{1}{2}+\frac{1}{6}=\frac{3}{6}+\frac{1}{6}=\frac{3+1}{6}=\frac{4}{6}=\frac{2}{3}$$

Example: Subtract $\frac{12}{15}-\frac{5}{16}$

Solution: Our first step is to find the LCD, so we will look at the prime factorizations of 15 and 16.

$$15=3\cdot 5 \text{ and } 16=2^4$$

The primes that we have are 2, 3, and 5. The highest power of 3 and 5 is 1 and the highest power of 2 is 4. This gives an LCD of

$$LCD=2^4\cdot 3\cdot 5=240$$

Next, we will find the equivalent fractions. For the first fraction, we must multiply top and bottom by 16 to get a denominator of 240 and for the second fraction, we must multiply top and bottom by 15 to get a denominator of 240.

$$\frac{12\cdot 16}{15\cdot 16}=\frac{192}{240} \text{ and } \frac{5\cdot 15}{16\cdot 15}=\frac{75}{240}$$

Now, we will subtract our fractions:

$$\frac{12}{15}-\frac{5}{16}=\frac{192}{240}-\frac{75}{240}=\frac{192-75}{240}=\frac{117}{240}=\frac{39}{80}$$

Multiplying Fractions

Steps

1. Multiply the numerators together
2. Multiply the denominators together
3. Reduce fractions

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$$

Example: $\frac{1}{3}\cdot\frac{4}{7}$

Solution: $\frac{1}{3}\cdot\frac{4}{7}=\frac{1\cdot 4}{3\cdot 7}=\frac{4}{21}$

Hint: It may be easier to cancel any like terms prior to multiplying. This will make the numbers smaller and easier to work with.

Example: $\frac{21}{55}\cdot\frac{11}{49}$

Solution: Notice: 11 and 55 are both divisible by 11 while 21 and 49 are both divisible by 7. We will cancel these like terms first.

$$\frac{21}{55}\cdot\frac{11}{49}=\frac{3}{5}\cdot\frac{1}{7}=\frac{3\cdot 1}{5\cdot 7}=\frac{3}{35}$$

Dividing Fractions

Steps: (KFC Rule)

1. **K**eep the first fraction as is.
2. **F**lip the second fraction (in other words take its reciprocal).
3. **C**hange the sign to multiplication.
4. Multiply the fractions together and reduce.

Example: $\frac{1}{3}\div\frac{4}{7}$ Solution: $\frac{1}{3}\div\frac{4}{7}=\frac{1}{3}\cdot\frac{7}{4}=\frac{1\cdot 7}{3\cdot 4}=\frac{7}{12}$

Note: When dividing fractions, you cannot reduce like terms until after you have completed the first 3 steps.

Example: $\frac{21}{55}\div\frac{35}{11}$ Solution: $\frac{21}{55}\div\frac{35}{11}=\frac{21}{55}\cdot\frac{11}{35}=\frac{3}{5}\cdot\frac{1}{5}=\frac{3\cdot 1}{5\cdot 5}=\frac{3}{25}$