Exponents

Exponents, square roots, and scientific notation

Exponents are shorthand for repeated multiplication of the same thing by itself. For instance, the shorthand for multiplying three copies of the number 5 is shown on the right-hand side of the "equals" sign in $(5)(5)(5) = 5^3$. The "exponent", 3 in this example, stands for however many times the value is being multiplied. The thing that's being multiplied, 5 in this example, is called the "base".

This process of using exponents is called "raising to a power", where the exponent is the "power". The expression "5^3" is pronounced as "five, raised to the third power" or "five to the third". There are two specially-named powers: "to the second power" is generally pronounced as "squared", and "to the third power" is generally pronounced as "cubed". So "5^3" is commonly pronounced as "five cubed".

When we deal with numbers, we usually just simplify; we'd rather deal with "27" than with "3^3". But with variables, exponents are needed, because we'd rather deal with "x^6" than with "*xxxxxx*".

Basic Rules of Exponents

1. *Addition Rule*: Whenever multiplying two terms with the same base, add the exponents:

$$(x^m)(x^n) = x^{(m+n)}$$

2. *Multiplication Rule*: Whenever an exponent expression that is raised to a power, multiply the exponent and power:

$$(x^m)^n = x^{mn}$$

3. *Subtraction Rule*: Whenever dividing two terms with the same base, subtract the exponents

$$\frac{x^m}{x^n} = x^{m-n}$$

4. *Negative Exponent Rule*: Whenever a term is raised to a negative power, place the term and the positive power on the other side of the fraction line.

$$x^{-n} = \frac{1}{x^n}$$

5. *Zero Power Rule*: Anything to the power zero is just "1".

$$m^0 = 1$$

6. *Distribution Rules:* Raising a product and a quotient to a power is the same as raising each term to the power first.

$$(xy)^n = x^n y^n \text{ and } \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

NOTE: The Distribution Rules only work over multiplication and division. They DO NOT work over addition and subtraction, which will be discussed later.

<u>**Examples**</u>

- **Simplify $(x^3)(x^4)$**

  To simplify this, think in terms of what those exponents mean. "To the third" means "multiplying three copies" and "to the fourth" means "multiplying four copies". Using this fact, expand the two factors, and then work backwards to the simplified form:

  $(x^3)*(x^4) = (xxx)*(xxxx)$
  $= xxxxxxx$
  $= \mathbf{x^7}$
  Note that x^7 also equals $x^{(3+4)}$. This demonstrates the first basic exponent rule.

- **Simplify $(y^3)(x^4)$**

  This can NOT be simplified because the bases are different: $(x^4)(y^3) = xxxxyyy = (x^4)(y^3)$. Nothing combines.

- **Simplify $(x^2)^4$**

  Just as with the previous example, think in terms of what the exponents mean. The "to the fourth" means that I'm multiplying four copies of x^2:
  $(x^2)^4 = (x^2)(x^2)(x^2)(x^2)$
  $= (xx)(xx)(xx)(xx)$
  $= xxxxxxxx$
  $= \mathbf{x^8}$
  Note that x^8 also equals $x^{(2\times 4)}$. This demonstrates the second exponent rule.

- **Simplify $(x\ y^2)^3$**

  If you have a product inside parentheses and a power on the parentheses, then the power goes on each element inside. Here, $(xy^2)^3 = (xy^2)(xy^2)(xy^2) = (xxx)(y^2y^2y^2) = (xxx)(yyyyyy) = x^3y^6 = (x)^3(y^2)^3$.

- **Simplify $(x/\,y)^2$**

  Here, the power goes on each element inside the parentheses.

  $$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$$

- **Simplify $[(3x^4y^7z^{12})^5\ (-5x^9y^3z^4)^2]^0$**

  Do not worry about that stuff inside the square brackets. The zero power on the outside means that the value of the entire thing is just **1**.

<u>Distribution Rule Cautions</u>

The Distribution Rule does NOT work if there is a sum or difference within a set of parentheses. Here, exponents do NOT distribute over the addition problem.

Examples:

- **Simplify $(3+4)^2$**

This equation does NOT simplify to $3^2 + 4^2 = 9 + 16 = 25$.

When in doubt, write out the expression according to the definition of the power. "Squared" means "times itself", so $(3 + 4)^2 = (3+4) * (3+4) = (7) * (7) = 49$

- **Simplify $(x\text{-}2)^2$**

Do NOT try to do this in your head.   Instead, write it out: "squared" means "times itself".
So $(x - 2)^2 = (x - 2)(x - 2) = xx - 2x - 2x + 4 = x^2 - 4x + 4$.

Mistakes are generally made when students try to do everything in their heads, instead of showing all their work. By doing things neatly and showing all of the work, this mistake can be avoided.

Negative Exponent Examples

Examples:

- **Write x^{-4} using only positive exponents.**

$$x^{-4} = \frac{1x^{-4}}{1} = \frac{1}{x^4}$$

- **Write x^2 / x^{-3} using only positive exponents.**

$$\frac{x^2}{x^{-3}} = \frac{1x^2}{1x^{-3}} = \frac{1x^2x^3}{1} = x^5$$

- **Write $2x^{-1}$ using only positive exponents.**

$$2x^{-1} = \frac{2x^{-1}}{1} = \frac{2}{x^1} = \frac{2}{x}$$

Note that the "2" above does not move with the variable; the exponent is only on the "*x*".

- **Write $(3x)^{-2}$ using only positive exponents.**

$$(3x)^{-2} = \frac{(3x)^{-2}}{1} = \frac{1}{(3x)^2} = \frac{1}{9x^2}$$

Unlike the previous exercise, the parentheses meant that the negative power did indeed apply to the three as well as the variable.

- **Write $(x^{-2} / y^{-3})^{-2}$ using only positive exponents.**

$$\left(\frac{x^{-2}}{y^{-3}}\right)^{-2} = \frac{(x^{-2})^{-2}}{(y^{-3})^{-2}} = \frac{(y^{-3})^2}{(x^{-2})^2} = \frac{y^{-6}}{x^{-4}} = \frac{x^4}{y^6}$$

This one can also be done as:

$$\left(\frac{x^{-2}}{y^{-3}}\right)^{-2} = \frac{(x^{-2})^{-2}}{(y^{-3})^{-2}} = \frac{x^4}{y^6}$$

Reference: http://www.purplemath.com/modules/exponent.htm